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Static Analysis of Historical Reinforced Masonry Walls under Horizontal Loads

D. Abruzzese[1] and G. Lanni [2]

ABSTRACT

Very often the horizontal resistance of an historical masonry building is incremented by using steel tie bars passing in the walls at the floor level.

In previous works the results for the cases of the walls with thin platbands or small "panels" have been presented. This paper presents a method and some results for the case in which the masonry panels between two piers are "deep".

1. INTRODUCTION

For a considerable number of masonry monumental buildings the main resistant structure is composed by a double array of orthogonal walls. Very often the walls of this regular pattern are not well connected each other, because the weak wooden floor does not allow the effective linking needed in presence of horizontal actions. In this case the "box effect" is guarantied by the steel tie bars passing at the floor level all along the walls.These tie bars, behind the control of the out-of-plane collapse for the walls, give a remarkable increment to the seismic resistance of each wall.

The first static effect is indeed on the prevention of an anticipated collapse of the weakest masonry piers, before all the wall were engaged with its total resistance. The steel ties allow the transferring of the horizontal loads to the strongest piers because the steel gives to the horizontal connections a traction resistance, missing in the platbands or in the masonry panels.

It has been already studied a mechanical model for the horizontal masonry connections consisting in simple monodimensional platbands (Abruzzese et al., 1992). This model has been used in order to show the collapse behaviour of the structure subjected to vertical fixed loads and horizontal gradually increasing loads. The study has been carried out in the framework of the limit analysis applied to the masonry structure (Heyman,1966) and then the piers and the platbands have been considered as a rigid, tensionless material, and the steel ties as elastic material, sustaining only traction. The study gave a simple algorithm to calculate the collapse multiplier and the stress in the ties, usually in the elastic limits.

A development of these studies has been the extension of the previous analysis to the cases in which the horizontal masonry connections, over the openings, are bidimensional masonry panels of rigid, tensionless material. This new model shows that steel ties, in the presence of the bidimensional connections, produce a different kinematic mechanism with two effects, new fractures in the piers and plasticity in the steel, with a remarkable increment of the collapse multiplier.

[1] Assistant professor , Dept. of Civil Eng., Univ. of Rome "Tor Vergata"
[2] Associate professor , Dept. of Civil Eng., Univ. of Rome "Tor Vergata"

A first tentative to develop a simple algorithm for the calculus of the collapse multiplier has been presented by Abruzzese et al.(1992), where the panels has been considered “narrow”, more similar to a beam from the geometric point of view. In that hypothesis the collapse multiplier for the wall without tie bars depends only on the geometry of the pier and not on that one of the panels. In this paper then we study the case of the “deep” panel, for which the collapse multiplier even for the case without tie bars depends strongly on the geometry of the panels.

2. PRELIMINARY ANALYSIS OF AN ELEMENTARY SCHEME

Let start with the study of the elementary scheme of a wall as shown in fig. 1, where G represent the total dead loads of the piers and λG are the resultants forces due to the seismic horizontal action.

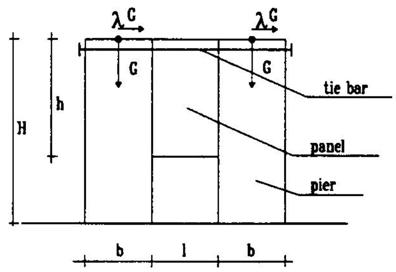

fig. 1

The collapse mechanism related with the geometric characteristics of the piers and panel can be that one shown in fig. 2a or in fig.2b. In the first case we call the panel “narrow”, while in the second case we call the panel “deep”. In both cases, because the reasons already discussed by Abruzzese et al. (1994), the tie bar yields and the total resistance of the system to the horizontal actions is due besides the uplifting of the weights G even to the plastic work performed by the yielding load T_o.

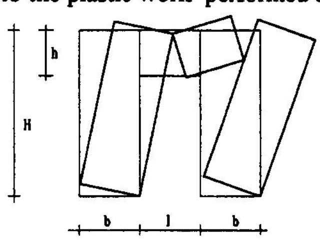

fig.2a

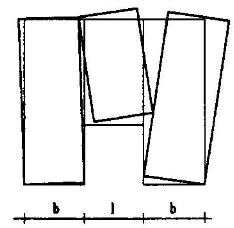

fig.2b

In order to assess if the structure behaviour is type a or b, we refer to the fig. 3a that shows, in the case of the “narrow” panel, the positions of the absolute and relatives centers C_i and C_{ij}. Let us point out the fact that if the center C_3 is on the upper part respect to the relative center C_{13}, a clockwise rotation of the pier 1 occurs, compatible with the position of the center C_1. It is clear that as the height h of the panel rises, the center C_3 goes down over the center C_{13}. When this happens it easy to verify that the rotation of the pier 1 becomes counter-clockwise and then anymore compatible with the position of the center C_1. In this case the panel can be called “deep”, and the position of the centers C_i and C_{ij} is that one shown in the fig. 3b, where it is possible to recognize that the distribution of the rotations of the piers and panel is compatible with the non-compenetration condition. In conclusion it is possible state that in the simple scheme of fig.1 the panel is defined "deep" if:

$$h > h^* = H\frac{l}{l+b} \qquad (1)$$

otherwise it is defined "narrow".

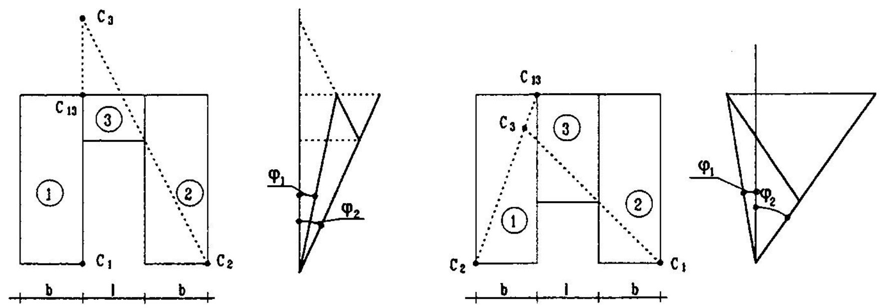

fig. 3a fig. 3b

The calculus of the collapse multiplier λ_0 can be performed by using the virtual work principle. In fact equalizing the active work of the horizontal thrusts to the resistant work developed from the weights and from the tensile stress in the tie bar, it is possible to obtain the equation leading to the collapse multiplier λ_0.

The simple examination of the horizontal displacements shown in fig. 3 allows us to express the values of the uplifting V of the weights G, of the horizontal displacement U of the thrusts λG and of the elongation Δ of the tie bar as a function of an unique kinematic parameter . That has been done in a previous work in the case of the "narrow" panel, and the method has given an expression quite simple of the collapse multiplier λ_0.

In the case of the "deep" panel, on the other hand, the expressions of the values V,U, Δ as a function of an unique kinematic parameter can be obtained in a slightly different way, in order to give useful results for the analysis of structural schemes more complex than that one shown in fig.1.

Let us assume the rotation φ_1 of the pier 1, positive if clockwise, as kinematic parameter. The displacements U_1 and V_1 (horizontal and vertical), related with the thrust and the weight acting on pier 1, can be expressed easily as:

$$U_1 = \varphi_1 H; \qquad V_1 = -\varphi_1 b/2 \qquad (2)$$

Moreover it is easy to calculate the displacements u_C and v_C of the point C belonging to the pier 1 and to the panel (point C_{13} in the fig. 3b):

$$u_C = \varphi_1 H; \qquad v_C = -\varphi_1 b \qquad (3)$$

It is now possible to calculate from the displacements u_C and v_C the displacement of any other point in the panel and in the pier 2. Let us then consider the scheme in fig.4, showing two masonry blocks 1 and 2 connected through the hinge B while the block 2 is connected to the ground with the hinge A.

The point C belonging to the block 1, because the kinematics, moves horizontally and vertically, with displacement components u_C and v_C.

The point D, center of the absolute rotation for the block 1, is characterized by the coordinate y_D, which, after few calculations, can be expressed in the following form:

$$y_D = H\ f(\rho,\beta,\nu). = H\,\xi \tag{4}$$

where:

$$\rho = h / H \qquad \beta = l / b \qquad \nu = v_C / u_C \tag{5}$$

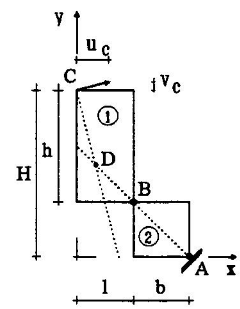

fig. 4

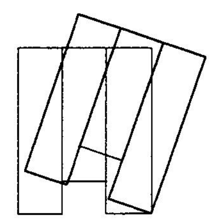

fig.5

The analysis of the horizontal displacements shown in fig.4 allows us to calculate in an easy way the rotations ω and γ of the two blocks:

$$\omega = u_C / H\ f'(\rho,\xi). = u_C / H\ q \tag{6}$$

$$\gamma = -\,u_C / H\ f''(\rho,\xi). = -\,u_C / H\ k \tag{7}$$

The (7), applied to the scheme in fig.3b, taking into account the (3), allows to evaluate the displacements U_2 and V_2 of the thrust λG and of the weight G corresponding to the pier 2:

$$U_2 = \varphi_2 H = -\,\varphi_1 k H; \qquad V_2 = \varphi_2 b/2 = -\,\varphi_1 k\, b/2 \tag{8}$$

while the tie bar elongation Δ is:

$$\Delta = U_2 - U_1 = -\,\varphi_1 H (k+1) \tag{9}$$

The collapse multiplier λ_0, after few calculations, can be written in the following way:

$$\lambda_0 = \frac{k+1}{k-1}\left(\frac{b}{2H} + \frac{T_0}{G}\right) \tag{10}$$

The (10) shows the increment of the collapse multiplier due to the presence of the tie bar and lights up the increment that, without tie bar, the collapse multiplier λ_0 presents as effect of the "deep" panel, compared with the value b/2H that we have with the "narrow" panel.

Moreover it is interesting to observe that if the dimension h of the panel becomes smaller, close to the limit value h*, given in (1), that distinguish the "deep" panel from the "narrow" one, the k factor defined in the (7), representing the ratio between the rotations φ_2 and φ_1:

$$k = |\varphi_2| / |\varphi_1| \tag{11}$$

tends to infinity and then the collapse multiplier λ_0 tends to the value within the commas in the (10), characteristic for the "narrow" panel schemes, as reported in a previous work.

On the other hand, if the dimension "h" rises, becoming equal to the limit value H, the factor k tends to 1 and the collapse multiplier λ_0, given from the (10), tends to infinity, being anymore statically admissible. In other words it should exist a limit dimension h** of the panel beyond which the collapse mechanism is that shown in fig.5 and the collapse multiplier λ_0 is:

$$\lambda_0 = \frac{2b+l}{2H} \tag{12}$$

3. THE MULTIPIER WALL

Let us now study the structural scheme in fig.6 in which more than two piers are present. On the basis of the considerations and results described in 2. it is possible to conclude that if the condition (1) is true then the collapse mechanism can be only that one shown in fig.7.

It is interesting to observe that in the two piers scheme the blocks acting in the mechanism correspond to those elements called piers and panels. On the contrary, in the multipier scheme only the piers at the extremities and the last panel on the right side are blocks of the mechanism, while any other pier breaks and splits in two parts and together with the panel on the left side becomes a whole block.

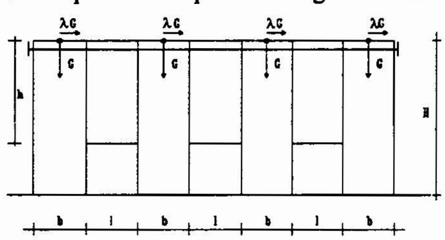

fig. 6

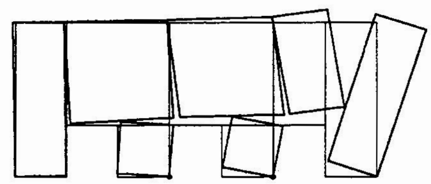

fig. 7

The mechanism shown in fig.7 can be considered as a sequence from left to right of kinematic schemes as that one shown in fig.4. In other words, since the rotation φ of the first pier on the left is assigned, it is possible to calculate the rotations of all the elements involved in the mechanism: in fact we can analyze any other scheme using the results obtained from the previous scheme on the left side.

Going in the details, if the N kinematic schemes are numbered from left to right, the rotation γ_i of the lower part of the pier and the rotation ω_i of the block composed by the upper part of the pier with the next left panel, are similar to the (6) and (7):

$$\omega_i = \frac{u_{Ci}}{H} q_i , \qquad \gamma_i = -\frac{u_{Ci}}{H} k_i \tag{13}$$

The displacements u_{Ci} and v_{Ci} that appear in the (13) can be calculated as function of the rotations ω_{i-1} and γ_{i-1} of the previous scheme:

$$u_{Ci} = u_{Ci-1} \qquad ; v_{Ci} = \gamma_{i-1}\, b - \omega_{i-1} b \tag{14}$$

After the calculation of the rotations ω_i and γ_i for i=1,...,N it can be state:

$$\omega_i = \varphi_0\, q^*_i \;; \qquad \gamma_i = -\,\varphi_0\, k^*_i \tag{15}$$

and then we can to calculate the horizontal displacements U_i of the thrusts for i=0,....,N, the uplifting V_i of the weights for i=0,....,N and the elongation Δ of the tie bar:

$$U_o = \varphi_0 H \;; \qquad U_i = \varphi_0 H \quad \text{for } i=1,\dots,N-1; \qquad U_N = -\varphi_0 H k^*_N \tag{16}$$

$$V_o = -\varphi_0 b/2; \qquad V_i = -\varphi_0 (q^*_i\, b/2 + k^*_i\, b) \quad \text{for } i=1,\dots,N; \qquad V_N = -\varphi_0 k^*_N\, b/2 \tag{17}$$

$$\Delta = -\,\varphi_0 H (k^*_N + 1) \tag{18}$$

Then, by applying the virtual principle work, the collapse multiplier λ_o can be obtained:

$$\lambda_0 = \frac{b}{2H}\, \frac{k^*_N + 1 + \sum_{i=1}^{N-1} q^*_i + 2k^*_i}{k^*_N - N} + \frac{T_0}{G}\, \frac{k^*_N + 1}{k^*_N - 1} \tag{19}$$

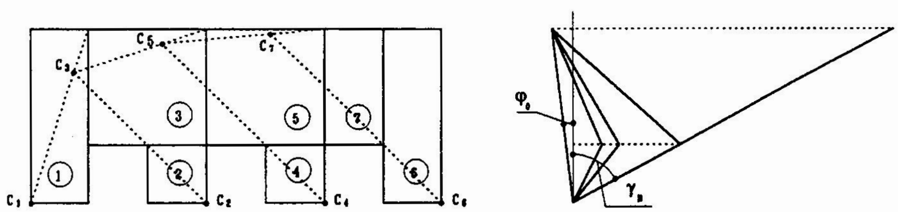

fig.8

From the fig.8, in which are drawn the absolute and relative center positions of the kinematics and the rotations φ_0 , ω_i , γ_i , it is possible to observe that:

$$k^*_N \ggg 1 \tag{20}$$

and then the (19) can be written in a simpler way:

$$\lambda_0 = \frac{b}{2H}\left(1 + \frac{\sum_{i=1}^{N-1}(q^*_i + 2k^*_i)}{k^*_N}\right) + \frac{T_0}{G} \tag{21}$$

4. THE MULTISTOREY WALL WITH A REGULAR ARRAY OF OPENINGS

Let us now study the structural scheme of a multistorey wall shown in fig.9. With the hypothesis of “deep” panels at each floor, the collapse mechanism is that one shown in fig.10. It is interesting to note that while at the first floor all ‘inner’ piers split in two parts (see prg. 3), at the upper levels this happens only partially. Indeed, if we call with index 0 the ‘outer’ left pier, with index N the ‘outer’ right one, with index i the ‘inner’ ones, with $i=1,\dots,N-1$, and then with index j the floor, it is easy to verify that at the j-th level the piers break from left side to the right one, till to the (N-j)-th position. On the other hand, for $i=N-j+1,\dots,N$ the pier remain unbroken in that floor.

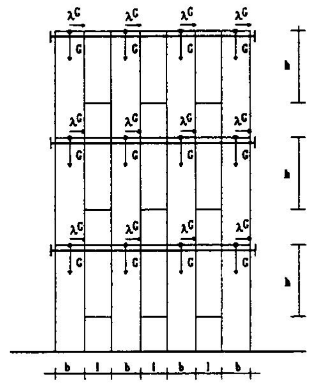

fig. 9

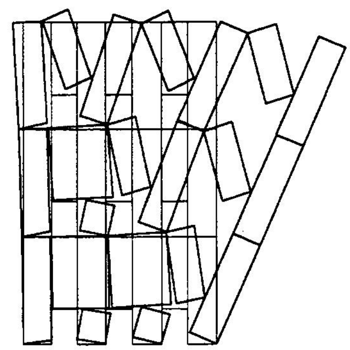

fig.10

In the fig. 10 it is possible to note that the mechanism at the j-th floor (except for the ‘outer’ piers), can be considered as a sequence, from left side to right side, of kinematic schemes quite similar to the one shown in fig. 4. Thus, once a rotation φ^*_{0j} of the external left pier is assigned , it is possible calculate, starting from left side, the sequence of the rotations γ_{ij} and ω_{ij} for all the blocks involved in the mechanism, as already explained in the prg.3. It is necessary to note that for $i \leq N-j$ (broken pier) γ_{ij} and ω_{ij} represent the rotation of the lower part of the pier and that one of the block formed by the upper part of the pier and the adjacent panel (fig.11a). On the other hand, for $i > N-j$ (unbroken pier) γ_{ij} and ω_{ij} represent the rotation of the pier and that one of the next panel (fig.11b).

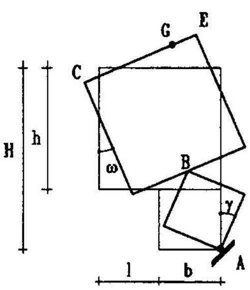

fig. 11a

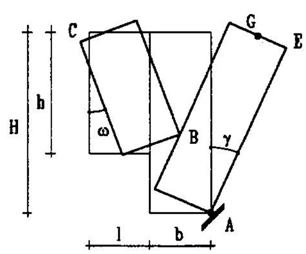

fig. 11b

Moreover, it is quite obvious that this procedure can be applied only after a preliminary analysis of the mechanism at the j-1-th level, because the displacements u_C and v_C in fig.4 represent the displacements of the point C related with the point A.

In order to improve the algorithm presented, it is useful to write:

$$u_{Ci,j} = u_{Ei-1,j} - u_{Ei,j-1} \qquad v_{Ci,j} = v_{Ei-1,j} - v_{Ei,j-1} \qquad (22)$$

where the point E is defined in the fig. 11a or fig. 11b, depending on the kinematic scheme (pier broken or not). We have:

$$\text{case a)} \quad u_E = \gamma\,(H-h) - \omega h + u_A; \qquad v_E = \gamma\, b - \omega b + v_A; \qquad (23)$$

$$\text{case b)} \quad u_E = \gamma\, H + u_A; \qquad v_E = v_A; \qquad (24)$$

Once the rotations ω_{ij} and γ_{ij}, for i=1,...,N, have been calculated and written in the form:

$$\omega_{ij} = \varphi^*_{0j}\, q^*_{ij} \qquad \gamma_{ij} = -\varphi^*_{0j}\, k^*_{ij} \qquad (25)$$

we obtain the rotation γ_{Nj} which must be equal to the rotation $\gamma_{N,1}$ because the "outer" left pier is unbroken all along the building (fig. 10). Then we can write the value of the rotation φ_{oj} of the "outer" left pier at the j-th floor:

$$\varphi_{oj} = -\gamma_{N1} / k^*_{Nj} \qquad (26)$$

and the (25) can be written:

$$\omega_{ij} = -\gamma_{N1} / k^*_{Nj}\, q^*_{ij} \qquad \gamma_{ij} = \gamma_{N1} / k^*_{Nj}\, k^*_{ij} \qquad (27)$$

By applying this procedure, starting from the first to the last floor of a M-level building, we obtain the rotations φ_{oj},, corresponding to the mechanism of the "outer" left pier, then the rotations ω_{ij} and γ_{ij}, where i=1,...,N-1 and j=1,...,M, corresponding to the mechanism of the "inner" part of the wall, and finally the rotation γ_{N1} of the "outer" right pier. It is then possible to calculate easily the horizontal displacements U_{ij} of the thrusts, the uplifting V_{ij} of the weights (displacements of the point G of fig.11) and finally the elongation Δ_j of the tie bars.

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